Scenario-based Model Predictive Control Approach for Heating Systems in an Office Building

Tomas Pippia, Jesus Lago, Roel De Coninck, Joris Sijs, and Bart De Schutter

Abstract— In the context of building heating systems control in office buildings, the current state-of-the-art applies either a deterministic Model Predictive Control (MPC) controller together with a nonlinear model, or a linearized model with a stochastic MPC controller. Deterministic MPC considers only one realization of the external disturbances, which can lead to a low performance solution if the forecasts of the disturbances are not accurate. Similarly, linear models are simplified representations of the building dynamics and might fail to capture some relevant behavior. In this paper, we improve upon the current literature by combining these two approaches, i.e. we adopt a nonlinear model together with a stochastic MPC controller. We consider a scenario-based MPC (SBMPC), where many realizations of the disturbances are considered, so as to include more possible future trajectories for the external disturbances. The adopted scenario generation method provides statistically significant scenarios, whereas so far in the current literature only approximate methods have been applied. Moreover, we use Modelica to obtain the model description, which allows to have a more accurate and nonlinear model. Lastly, we perform simulations comparing standard MPC vs SBMPC vs an optimal control approach with measurements of the external disturbances, and we show how our proposed scenario-based MPC controller can achieve a better performance compared to standard deterministic MPC.

Index Terms— Model predictive control, Building automation, Building heating systems, Scenario-based control

I. INTRODUCTION

A. Motivation

Heating systems represent more than half of the total energy usage in buildings, which in turn accounts for around 40% of the total energy use [1], [2]. In building heating systems it is important not only to try to reduce the consumed energy, but also to reduce as much as possible the discomfort caused to occupants. In order to properly control the room temperature in a building, additional information should be included, e.g. external temperature, solar irradiance, occupancy of the building. However, most of the currently implemented control strategies in real buildings are simple rule-based algorithms that are not very efficient and include only current measurements of the aforementioned information,

but not predictions. In this regard, Model Predictive Control (MPC) stands out as suitable control tool that can be applied efficiently to building heating systems, since it can naturally include constraints related to the heat producing devices and also to the comfort of the occupants [3]–[10]. Moreover, since in the MPC framework the control problem is turned into an optimization one, MPC allows to optimize a cost function defined in terms of economic costs and discomfort.

In the presence of external disturbances, as is the case for building heating systems, two different MPC strategies can be used, namely robust MPC and stochastic MPC [11]. While robust strategies guarantee that the constraints are satisfied for every possible realization of the disturbance, stochastic MPC approaches consider a relaxation of the constraints and allow a mild constraint violation in the system. Which of the two strategies is more suitable for a certain system depends on the application. For the case of building heating systems control, the stochastic approach is generally preferred, as a robust solution would be too conservative [4], [12]. Indeed, the main constraints in building heating systems are related to the comfort bounds, which can be safely violated without causing danger to people or to the equipment, as long as the violation is not too high and does not last for a long time.

B. Main Contribution

In our work, we focus on a stochastic MPC application to an office building. Due to the fact that the model of the system is nonlinear, as stressed in [12], we adopt a randomized approach, namely Scenario-Based MPC (SBMPC). The main idea of this approach is to consider a certain number of disturbance realizations, i.e. scenarios, and to satisfy the constraints while minimizing the average cost for each of the scenarios. SBMPC is able to deal well with both nonlinear systems and probability distributions obtained empirically, making it a very suitable tool for a building heating control problem [12]. Data from measurements of several disturbances, e.g. outdoor ambient temperature, solar irradiance, was provided by [13]. Thus, although this paper uses a simulation model, we make a comparison between different controllers using real disturbances data. Furthermore, unlike many works in the current literature, where a nonlinear state space equation model is usually linearized, e.g. [4]–[7], we implement the model through the language Modelica [14], [15], because it can provide a much more reliable and close to reality model. Other works, e.g. [8]–[10], have implemented MPC by developing the model through Modelica but, to the best of our knowledge, none of these works has implemented stochastic MPC in a Modelica

framework. Therefore, due to the usage of Modelica, we do not approximate our model to a linear one and instead we use the resultant nonlinear model for control purposes.

In addition to using a nonlinear model, this article also improves the existing literature by providing a method to generate scenarios that are more realistic. In particular, while different scenario generation methods have been proposed in the literature of SBMPC applied to buildings [4], [6], [7], they all have several disadvantages when generating realistic scenarios. Indeed, when generating scenarios of time series, it is paramount that the values of a single scenario are correctly correlated, i.e. the scenarios cannot simply be generated using the marginal distributions of the different time points or prediction horizons [16]. In this context, [4] and [7] propose a method for generating scenarios that uses an empirical copula. While the method attempts to capture the correlation between the different prediction time steps ahead, i.e. the time points where the scenarios are defined, it has three drawbacks: 1) it assumes that the marginal distributions only depend on the prediction time steps but not on time itself, e.g. while the distribution of 1 hour ahead is different than 2 hours ahead, the method assumes that the n hours ahead distribution is the same at any hour of the day and at any day of the week; 2) it considers that these distributions are stationary, e.g. the distributions are the same in summer and in winter; 3) it builds marginal distributions based solely on historical data, e.g. it builds temperature scenarios independently from weather forecasts. Another algorithm presented in [6] tries to overcome one limitation of the previous method: by using an analytical copula method, it generates scenarios that are not only correlated between prediction time steps but also in time itself. However, unlike the others, it builds the marginal distributions using historical data and assumes that these distributions are stationary. With this motivation, in this paper we try to overcome the shortcomings of the existing literature by using the a parametric Gaussian copula method. It is based on copula theory like the other methods; however, unlike the former ones, the method is able to capture non-stationary relations, it models correlations not only between prediction time points but also between time itself, and it uses real forecasts to build the marginal distributions.

This paper is thus the next step of [10], where a deterministic MPC algorithm was proposed using a Modelica building model. The main contribution of our work is threefold:

- we propose, for the first time, a building heating systems control method that considers a Modelica nonlinear model and an SBMPC controller;
- we generate scenarios for the SBMPC controller using statistically significant scenarios rather than the more naive ones usually adopted in the literature;
- we perform a comparison between standard deterministic MPC and SBMPC.

C. Outline

The outline of the article is as follows. We present our problem in Section II. The controller applied to the system
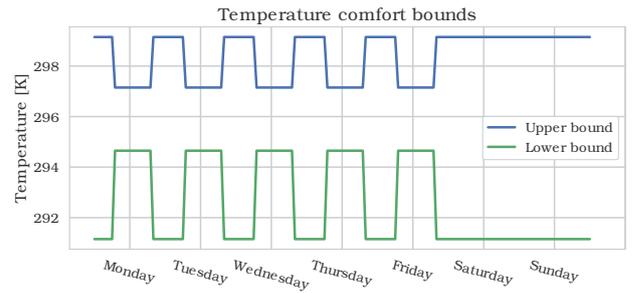

Fig. 1.   Comfort bound profiles.

and the controllers compared in the simulation section are presented in Section III. We present the scenario generation method used in the SBMPC controller in IV. Section V is devoted to present and to discuss the results of the simulations and lastly conclusions and remarks for future work are discussed in Section VI.

II.   PROBLEM DESCRIPTION

A. Office Buildings

We consider large office buildings with local heat production units. The thermal comfort bounds are set to 21.5°C and 24°C during occupation hours and 18°C and 26°C during the non-occupation hours. Figure 1 shows the profile of the temperature comfort bounds.

Measurements from external disturbances are available. Disturbances include the external temperature, the solar irradiance, the ventilation profile, and the domestic hot water. Unfortunately, we have no measure of the occupancy, since it is not easy to measure [17]. Therefore, we assume that the occupation profile is fixed. During "standard" working hours, we assume the building is fully occupied, while outside of these hours we assume it is empty.

The building has 2 control inputs, i.e. $\boldsymbol{u} = \begin{pmatrix} Q_{\mathrm{hea}} & Q_{\mathrm{coo}} \end{pmatrix}^{\top}$, where Q_{hea} is the amount of heating power transfered to the building and Q_{coo} is the cooling power provided to the building. The quantity Q_{hea} contains the heat produced by the two gas boilers. The model of the building is based on an RC-model, which has been identified through the Grey-Box Buildings toolbox [18].

B. Modelica

The overall model, including the heating, cooling, and ventilation units is obtained using Modelica [14], [15], an object-oriented language designed to model the behavior of complex physical systems. One of the main features of Modelica is the capability of describing a system by differential-algebraic equations, without the need of transforming the model into an ordinary-differential equation representation.

The advantage of using Modelica in our approach is that we can improve the precision and the amount of detail of the model. Indeed, while many other works [4]–[7] use a linearization of a nonlinear model, in this work we directly use a nonlinear model and therefore we have a more

meaningful simulation of the real building. For more details on the modeling procedure of buildings through a Modelica environment, the interested reader is referred to [10].

C. Practical Implementation in Buildings

The overall automated control scheme of the MPC framework is shown in Figure 2. The several steps taken by the building energy control and management system are [10]:

1) Monitoring: the building energy control and management system monitors the rooms and performs some measurements, e.g. water temperature, heat flux. Moreover, an independent system consists of calorimeters for each of the circuits;

2) Data collection: weather forecasts are obtained from [13];

3) State estimation: as not all the states can be measured, some of them, e.g. internal wall temperatures, have to be estimated. As explained in [10], we estimate only the initial states for a model with given parameters, instead of estimating the parameters of a model that minimize the residuals of the measurements;

4) Optimal Control Problem (OCP): the OCP, explained in Section III, is solved at every time step, with a control sampling time of 1h. The first inputs of the optimal sequence are applied to the system, such that every 5 minutes the optimal control trajectories computed in the last OCP are interpolated and sent to the building. This is done in order to have an updated control action, while reducing the computational complexity by having a sampling time of 1h.

Simulating the behavior of the system requires the same steps, with the difference that instead of applying the inputs to the real building and sampling the new values of states, we apply the inputs to a model of the system and simulate it for one time step.

The OCP in step 4 is solved through JModelica.org [19]. The direct collocation method is used to discretize time and by doing so the optimization problem is reduced to nonlinear programming problem [20]. CasADi [21] is used to obtain the first-order and second-order derivatives of the expressions in the nonlinear programming problem with respect to the decision variables, required by the solvers used by JModelica.org. As nonlinear programming problem solver, we use IPOPT [22] together with the sparse linear solver MA86 [23].

III. CONTROLLER SCHEME

A. Standard MPC

MPC is an established and well-known control tool that has been studied since the '70s [11], [24], [25]. Moreover, it has successfully been applied to building heating systems [3]–[10]. The main idea behind MPC is to convert a control problem into an optimization problem, and this allows to naturally include constraints and a cost function into the optimization problem. When applied to building heating systems, the cost function is usually a weighted sum of two terms, i.e. an energy cost and a discomfort cost, and the
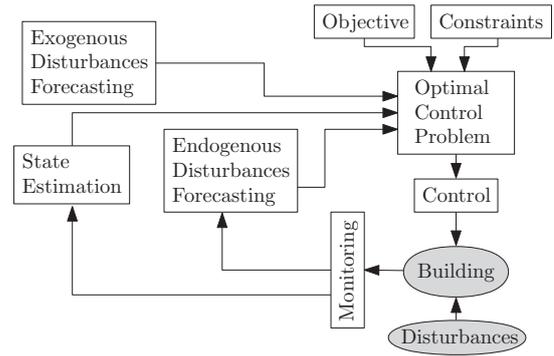

Fig. 2. Scheme of the MPC framework [10].

goal is to minimize these two objectives. The weight acts as trade-off between the two objectives. In standard MPC, the disturbances are assumed to be perfectly known, i.e. only one realization of the disturbances is considered and it is assumed that the forecast is perfect. This limitation could bring to a decrease in performance and therefore a SBMPC approach is expected to improve the performance.

B. Scenario Based MPC

In SBMPC many realizations - or scenarios- of the disturbances are considered, and the cost function is the average of the costs for each scenario, while the input vector is the same for all the scenarios. In this way, it is possible to include a wider range of possible disturbances and achieve a better performance compared to standard MPC. Indeed, it might happen that the only scenario considered by a standard MPC technique is too far from the real one, which decreases the performance. SBMPC strategy is recommended when the model of the system is nonlinear and there is no *a priori* information on the shape of the disturbance [12]. The algorithm that we use to generate the scenarios is based on a Gaussian copula method and explained in Section IV.

C. Cost Function and MPC Implementation

The cost function we consider is a weighted sum of two different objectives:

$$J = J_\mathrm{e} + \alpha J_\mathrm{d}, \tag{1}$$

where J_e represents the energy cost, J_d represents the thermal discomfort cost, and α is a weighting factor. These two costs are computed as:

$$J_\mathrm{e} = \sum_{i=1}^{N} c_\mathrm{g} E_\mathrm{g}(i) + c_\mathrm{e} E_\mathrm{e}(i) \tag{2}$$

$$J_\mathrm{d} = \sum_{i=1}^{N} \theta_\mathrm{occ}(i) \left[\max \left(T_\mathrm{zon}(i) - T_\mathrm{max}^\mathrm{z}(i), 0 \right) + \right.$$
$$\left. \min \left(T_\mathrm{zon}(i) - T_\mathrm{min}^\mathrm{z}(i), 0 \right) \right]^2, \tag{3}$$

where i represents the time steps, N is the prediction horizon, c_g and c_e represent the gas and electricity price, respectively,

E_g and E_e represent the gas and electricity consumption, respectively, T_zon is the averaged temperature in the rooms, T_min^z and T_max^z represent the minimum and maximum temperature comfort bounds, respectively, and $\theta_\text{occ} = 1$ during occupation hours and 0 elsewhere. The cost J_d acts as a soft constraint, i.e. it penalizes the deviations of the temperature outside of the temperature comfort bounds. In this way, it is possible for the controller to allow a violation of the thermal constraints if this leads to a lower total cost.

At each time step, the MPC problem is solved, yielding the optimal control input sequence $\boldsymbol{u}^*$. Then, only the first element of the sequence is applied, the horizon is moved one time step forward, and the optimization problem is solved again.

IV. SCENARIO GENERATION

The most important step in the SBMPC is generating the scenarios for the optimal control problem and these scenarios should be statistically significant. This means that it is not enough to take a single forecast, or measurement, and apply some noise to it, but rather we need a tool that reflects the variation in the noise in different prediction time steps and also the correlation between two consecutive predictions. In the context of buildings and SBMPC, some works from the literature have tried to address some of these issues [4], [6], [7]. Nevertheless, as mentioned in the introduction, the existing methods have 2 drawbacks: they consider that the distribution used to sample scenarios is stationary and that it can be built solely based on historical data, e.g. they incorrectly assume that temperature scenarios are independent of temperature forecasts. When considering the type of scenarios needed in the context of SBMPC and buildings, e.g. temperature or solar irradiance, it is clear that: 1) the marginal distributions of these scenarios, i.e. the probabilistic distribution of the variable of interest at each time prediction time step, cannot be stationary; 2) the marginal distributions should be based on weather forecasts to account for the latest available information, i.e. modeling the marginals based on historical values will likely underestimate or overestimate the real values.

In our work, to address the issues of the existing methods, we use a parametric Gaussian copula method that is able to capture non-stationary relations, as it models correlations across prediction time steps and time itself, and it uses real forecasts to build the marginal distributions. It is important to note that, while this is the first time the method is used in the context of buildings, it has successfully been used before in the context of wind forecasting [16].

A. Methodology

The classical setup in scenario generation for time series is the following: at time k and for a given horizon N, we need to generate predictive scenarios for the random variables $X_{k+1}, \ldots, X_{k+N}$ using the marginal cumulative distribution functions (CDFs),

$$F_j(x_j) := P(X_{k+j} < x_j), \quad \forall j = 1, \ldots, N.$$

These marginal distributions are given as they can easily be obtained using some forecasting technique, e.g. weather based forecasts or quantile regression based on historical data. In this context, the main issue to generate scenarios is that we cannot simply sample from these marginal CDFs as the samples would be uncorrelated with each other. Indeed, to generate realistic scenarios, one should sample instead from the full CDF, i.e. $F(x_1, \ldots, x_N) = P(X_{k+1} < x_1, \ldots, X_{k+N} < x_N)$.

The method that we consider in this paper uses copula theory to build the full CDF and then to sample from it. In detail, Sklar's theorem [26] states that every multivariate cumulative distribution function $F(x_1, \ldots, x_N) = P(X_1 < x_1, \ldots, X_N < x_N)$ of a random vector $[X_1, \ldots, X_N]$ can be expressed in terms of its marginals $F_j(x_j) = P(X_j < x_j)$ using a copula function C:

$$F(x_1, \ldots, x_N) = C(F_1(x_1) \ldots, F_N(x_N)). \quad (4)$$

Based on this theorem, the considered method builds a Gaussian copula based on the marginal CDFs obtained from forecasts. In addition, it uses a covariance matrix Σ_k that is updated online. These two properties of the method make it valuable to our application: it generates scenarios based on forecasts and not historical data and it uses a distribution to generate scenarios that is non-stationary and that it is adapted on time. For details on the method and how to generate the copula function $C(\cdot)$ from the marginals we refer to [16].

B. Generating scenarios

For our work, the marginal CDFs of the solar irradiance are built using a point forecast that considers weather information (as done in [27]), and then using quantile regression to build the CDFs of the errors of the point forecasts. For the temperature, we use the same procedure. After generating these marginal CDFs, we simply build the copula function and sample scenarios from it. An example of the external temperature scenarios generated with the method presented in this section is shown in Figure 3, where we also show in black color the real measurement. It can be seen that at the beginning the values of the scenarios are close to each other, but as the prediction horizon increases they are more dispersed. Moreover, the generated scenarios stay around the real measurement.

V. CASE STUDY

In this section we compare the results of the simulations carried out in the winter season with three different methods, i.e. perfect information optimal control problem (PI-OCP), which uses the measurements of the disturbances, the standard deterministic MPC strategy (Det-MPC), and SBMPC. Note that PI-OCP involves an optimal control problem run for the whole duration of the simulation and it yields the best achievable performance if the real disturbance values are considered.

The building that we consider for simulations is an office building in Brussels, Belgium, with 7 floors and a total surface of 10000 m^2. The heating system consists of 2
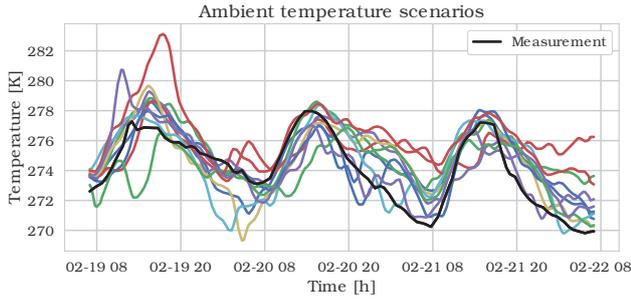
Fig. 3. Example of 10 ambient temperature scenarios generated with the parametric Gaussian copula method presented in Section IV. The real measurement is shown in black color.
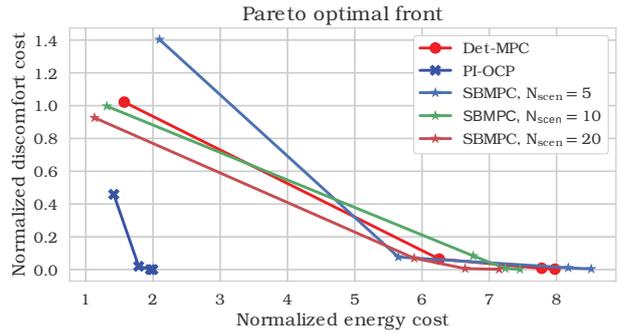

Fig. 4. Pareto optimal front with different number of scenarios and values of α, i.e. $\alpha \in \{1, 10, 100, 1000\}$.

TABLE I

NORMALIZED TOTAL COSTS FOR SIMULATIONS REPORTED IN FIGURE 4. FOR EACH COLUMN, THE MINIMUM COST BETWEEN THE DET-MPC AND THE SBMPC VALUES IS HIGHLIGHTED IN BOLD.

	$\alpha = 1$	$\alpha = 10$	$\alpha = 100$	$\alpha = 1000$
PI-OCP	1.875	1.991	2.007	2.011
Det-MPC	2.594	6.897	8.604	10.660
SBMPC, $N_{scen} = 5$	3.500	**6.415**	9.288	12.860
SBMPC, $N_{scen} = 10$	2.311	7.600	8.012	9.498
SBMPC, $N_{scen} = 20$	**2.057**	6.581	**7.278**	**8.987**

gas boilers of 400 kW each and 1 chiller of 400 kW. Moreover, the building is occupied during weekdays and not on weekends. The value of the prices in (2) are $c_g = 0.04€/kWh$ and $c_e = 0.15€/kWh$ [10].

Figure 4 shows the Pareto optimal front for PI-OCP, Det-MPC, and SBMPC for different values of the number of scenarios, N_{scen}. The four points for each curve correspond to four different values of α in (1), i.e. $\alpha \in \{1, 10, 100, 1000\}$; the leftmost point in each curve corresponds to $\alpha = 1$ and the three successive points in the curve correspond respectively to $\alpha = \{10, 100, 1000\}$. The values of the costs are the average of several simulations and are normalized to the number of total simulation hours in each simulation. From this figure we can therefore see how each different strategy manages the trade-off between discomfort and energy costs. First of all, we can notice how PI-OCP yields much lower costs than the other strategies, but this is expected since this strategy knows exactly the value of the disturbances and has a larger horizon. Also, we can notice that many points of the SBMPC curves are skewed to the upper-left part, compared to the Det-MPC curve. This means that the SBMPC strategy is better at minimizing the energy costs J_e, while providing a similar comfort, compared to the Det-MPC case. Notice also that as α increases, the soft comfort constraints become tighter and therefore for all the strategies the discomfort cost becomes very low, while the energy cost increases. This is due to the fact that in order to satisfy the tighter comfort bounds, more energy has to be used in the heating system.

The results of these simulations are also shown in Table I. For each column, the minimum cost between Det-MPC and the SBMPC strategies is highlighted in bold, in order to show which strategy performed better between Det-MPC and SBMPC. Note that, for all the strategies, the total cost becomes higher as α increases. This is related to the fact that with a larger α, the controller has to spend more energy to satisfy the comfort bounds. For what concerns the comparison between SBMPC and Det-MPC, it can be observed from Table I that in 3 out of 4 cases SBMPC performs worse than Det-MPC for $N_{scen} = 5$, but already for $N_{scen} = 10$ SBMPC outperforms Det-MPC.

Figures 5 and 6 show the results of a representative simulation for $\alpha = 100$ and $N_{scen} = 10$. We can notice from Figure 5 that the evolution of the room temperature in the three cases is quite similar. In particular, the Det-MPC solution and the SBMPC one have a similar evolution. On the other hand, since PI-OCP has perfect information about the disturbances and since it has a horizon equal to the duration of the simulation, it can compute a control action that allows the room temperature to stay within the comfort bounds for most of the time. In Figure 6 we show the evolution of the control input[1] Q_{hea}. Notice that also for this variable the evolution is quite similar between the three different cases. In particular, there are some slight differences between the solution provided by Det-MPC and by SBMPC, which, overall, leads to a lower cost of the SBMPC solution. Indeed, for this specific simulation, the normalized[2] J_e cost is 8.087 for Det-MPC and 7.238 for SBMPC, while the normalized J_d cost is 0.00924 for Det-MPC and 0.00774 for SBMPC. Given that $\alpha = 100$ in this case, the total cost is 9.0126 for the Det-MPC and 8.012 for SBMPC. Therefore, we can claim that the proposed SBMPC framework has a better performance compared to the current state-of-the-art controller.

VI. CONCLUSIONS

We have presented an application of stochastic scenario-based model predictive control to building heating in office buildings, performing simulations with real data. The paper shows the advantages of using an SBMPC approach compared to a standard, deterministic MPC approach. In

---

[1]Since we are considering a simulation in winter, Q_{coo} is always null.

[2]Normalized with respect to the total number of simulation hours, i.e. the cost shown is a "per hour" cost.
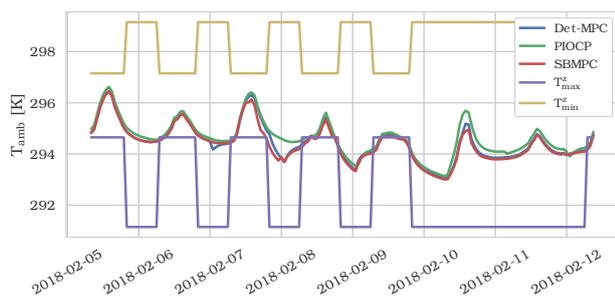
Fig. 5. Simulation profile of the room temperature evolution of one winter week with the three different methods considered in Section V. $T_{\max}^z$ and $T_{\min}^z$ stand for the maximum and minimum comfort temperature, respectively.
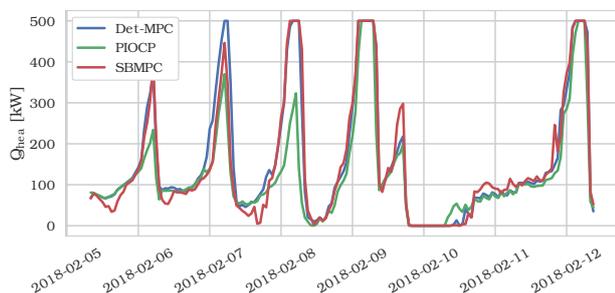

Fig. 6. Simulation results of the heating power profile of one winter week with the three different methods considered in Section V.

particular, due to the fact that the stochastic controller can include multiple realizations of the disturbances, it is able to respond better to deviations in outside temperature or solar irradiance.

In future work, experiments on the real building in the winter season and comparison of SBMPC vs standard MPC will be performed as a continuation of this work. Moreover, we plan to carry out an in depth analysis on how the performance of SBMPC is influenced by the kind of scenario generation algorithm chosen.

REFERENCES

[1] L. Pérez-Lombard, J. Ortiz, and C. Pout. A review on buildings energy consumption information. *Energy and Buildings*, 40(3):394–398, 2008.
[2] European environment agency. Final energy consumption by sector and fuel. https://www.eea.europa.eu/data-and-maps/indicators/final-energy-consumption-by-sector-9/assessment-1, 2016.
[3] Z. Váňa, J. Cigler, J. Široký, E. Žáčeková, and L. Ferkl. Model-based energy efficient control applied to an office building. *Journal of Process Control*, 24(6):790–797, 2014. Energy Efficient Buildings Special Issue.
[4] A. Parisio, M. Molinari, D. Varagnolo, and K. H. Johansson. A scenario-based predictive control approach to building HVAC management systems. In *2013 IEEE International Conference on Automation Science and Engineering (CASE)*, pages 428–435, 2013.
[5] F. Oldewurtel, A. Parisio, C. N. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari. Use of model predictive control and weather forecasts for energy efficient building climate control. *Energy and Buildings*, 45:15–27, 2012.

[6] Y. Long, S. Liu, L. Xie 1, and K. H. Johansson. A scenario-based distributed stochastic MPC for building temperature regulation. In *2014 IEEE International Conference on Automation Science and Engineering (CASE)*, pages 1091–1096, 2014.
[7] A. Parisio, D. Varagnolo, D. Risberg, G. Pattarello, M. Molinari, and K. H. Johansson. Randomized model predictive control for HVAC systems. In *Proceedings of the 5th ACM Workshop on Embedded Systems For Energy-Efficient Buildings*, BuildSys'13, pages 19:1–19:8. ACM, 2013.
[8] F. Jorissen, W. Boydens, and L. Helsen. TACO, an automated toolchain for model predictive control of building systems: implementation and verification. *Journal of Building Performance Simulation*, 12(2):180–192, 2019.
[9] J. Drgoňa, D. Picard, M. Kvasnica, and L. Helsen. Approximate model predictive building control via machine learning. *Applied Energy*, 218:199–216, 2018.
[10] R. De Coninck and L. Helsen. Practical implementation and evaluation of model predictive control for an office building in Brussels. *Energy and Buildings*, 111:290–298, 2016.
[11] D. Q. Mayne. Model predictive control: Recent developments and future promise. *Automatica*, 50(12):2967–2986, 2014.
[12] M. Farina, L. Giulioni, and R. Scattolini. Stochastic linear model predictive control with chance constraints – a review. *Journal of Process Control*, 44:53–67, 2016.
[13] 3E, Kalkkaai 6 Quai à la Chaux, 1000 Brussels, Belgium. http://www.3e.eu/.
[14] The Modelica Association. https://www.modelica.org.
[15] P. Fritzson. *Principles of Object-Oriented Modeling and Simulation with Modelica 3.3: A Cyber-Physical Approach*. IEEE Press. Wiley, 2 edition, 2015.
[16] P. Pinson, H. Madsen, H. A. Nielsen, G. Papaefthymiou, and B. Klöckl. From probabilistic forecasts to statistical scenarios of short-term wind power production. *Wind Energy*, 12(1):51–62, 2008.
[17] T. Labeodan, W. Zeiler, G. Boxem, and Y. Zhao. Occupancy measurement in commercial office buildings for demand-driven control applications—a survey and detection system evaluation. *Energy and Buildings*, 93:303–314, 2015.
[18] R. De Coninck, F. Magnusson, J. Åkesson, and L. Helsen. Toolbox for development and validation of grey-box building models for forecasting and control. *Journal of Building Performance Simulation*, 9(3):288–303, 2016.
[19] J. Åkesson, K.-E. Årzén, M. Gäfvert, T. Bergdahl, and H. Tummescheit. Modeling and optimization with Optimica and JModelica.org—Languages and tools for solving large-scale dynamic optimization problems. *Computers & Chemical Engineering*, 34(11):1737–1749, 2010.
[20] F. Magnusson and J. Åkesson. Collocation Methods for Optimization in a Modelica Environment. In *Proceedings of the 9th International MODELICA Conference; September 3-5; 2012; Munich; Germany*, number 76, pages 649–658. Linköping University Electronic Press; Linköpings Universitet, 2012.
[21] J. A. E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl. CasADi – A software framework for nonlinear optimization and optimal control. *Mathematical Programming Computation*, In Press, 2018.
[22] A. Wächter and L.T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, 2006.
[23] HSL(2013). A collection of Fortran codes for large scale scientific computation. http://www.hsl.rl.ac.uk.
[24] J. M. Maciejowski. *Predictive Control with Constraints*. Prentice Hall, England., 2002.
[25] E. F. Camacho and C. B. Alba. *Model Predictive Control*. Advanced Textbooks in Control and Signal Processing. Springer London, 2013.
[26] A. Sklar. Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut Statistique de l'Université de Paris*, 8:229–231, 1959.
[27] J. Lago, K. De Brabandere, F. De Ridder, and B. De Schutter. Short-term forecasting of solar irradiance without local telemetry: A generalized model using satellite data. *Solar Energy*, 173:566–577, 2018.